MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2016**

1 hour 15 minutes

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

This document consists of **4** printed pages and **1** insert.

1 The random variable X has the distribution Po(3.5). Find P$(X < 3)$. [3]

2 Dominic wishes to choose a random sample of five students from the 150 students in his year. He numbers the students from 1 to 150. Then he uses his calculator to generate five random numbers between 0 and 1. He multiplies each random number by 150 and rounds up to the next whole number to give a student number.

(i) Dominic's first random number is 0.392. Find the student number that is produced by this random number. [1]

(ii) Dominic's second student number is 104. Find a possible random number that would produce this student number. [1]

(iii) Explain briefly why five random numbers may not be enough to produce a sample of five student numbers. [1]

3 A men's triathlon consists of three parts: swimming, cycling and running. Competitors' times, in minutes, for the three parts can be modelled by three independent normal variables with means 34.0, 87.1 and 56.9, and standard deviations 3.2, 4.1 and 3.8, respectively. For each competitor, the total of his three times is called the race time. Find the probability that the mean race time of a random sample of 15 competitors is less than 175 minutes. [5]

4 The manufacturer of a tablet computer claims that the mean battery life is 11 hours. A consumer organisation wished to test whether the mean is actually greater than 11 hours. They invited a random sample of members to report the battery life of their tablets. They then calculated the sample mean. Unfortunately a fire destroyed the records of this test except for the following partial document.

Test of the mean batter the table	
Sample size, n	
Sample mean (hours)	11.8
Is the result significant at the 5% level?	Yes
Is the result significant at the 2.5% level?	No

Given that the population of battery lives is normally distributed with standard deviation 1.6 hours, find the set of possible values of the sample size, n. [5]

5 It is claimed that 30% of packets of Froogum contain a free gift. Andre thinks that the actual proportion is less than 30% and he decides to carry out a hypothesis test at the 5% significance level. He buys 20 packets of Froogum and notes the number of free gifts he obtains.

    (i) State null and alternative hypotheses for the test. [1]

    (ii) Use a binomial distribution to find the probability of a Type I error. [5]

Andre finds that 3 of the 20 packets contain free gifts.

    (iii) Carry out the test. [2]

6 A variable X takes values 1, 2, 3, 4, 5, and these values are generated at random by a machine. Each value is supposed to be equally likely, but it is suspected that the machine is not working properly. A random sample of 100 values of X, generated by the machine, gives the following results.

$$n = 100 \qquad \Sigma x = 340 \qquad \Sigma x^2 = 1356$$

   **(i)** Find a 95% confidence interval for the population mean of the values generated by the machine. [6]

    (ii) Use your answer to part **(i)** to comment on whether the machine may be working properly. [2]

7 Men arrive at a clinic independently and at random, at a constant mean rate of 0.2 per minute. Women arrive at the same clinic independently and at random, at a constant mean rate of 0.3 per minute.

    (i) Find the probability that at least 2 men and at least 3 women arrive at the clinic during a 5-minute period. [4]

    (ii) Find the probability that fewer than 36 people arrive at the clinic during a 1-hour period. [5]

[Question 8 is printed on the next page.]

8
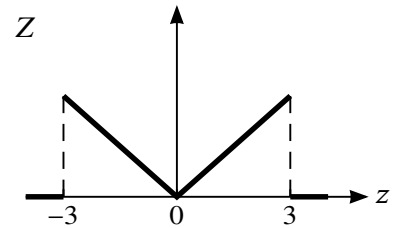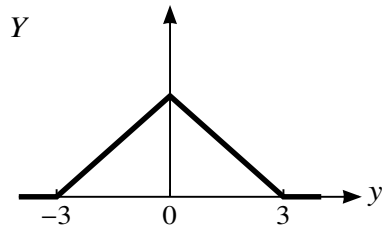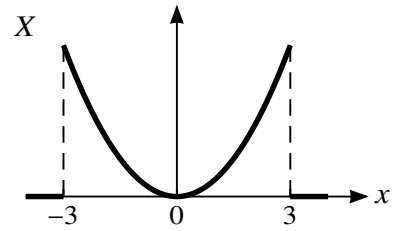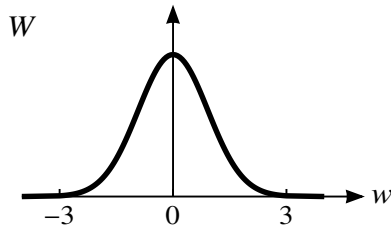
The diagrams show the probability density functions of four random variables W, X, Y and Z. Each of the four variables takes values between -3 and 3 only, and their standard deviations are σ_W, σ_X, σ_Y and σ_Z respectively.

(i) List σ_W, σ_X, σ_Y and σ_Z in order of size, starting with the largest. [2]

(ii) The probability density function of X is given by

$$f(x) = \begin{cases} \frac{1}{18}x^2 & -3 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $\sigma_X = 2.32$ correct to 3 significant figures. [3]

(b) Calculate $P(X > \sigma_X)$. [3]

(c) Write down the value of $P(X > 2\sigma_X)$. [1]